ARM ANGLE AND JUMP FORCE ON BASKETBALL SHOT DISTANCE

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ABSTRACT

Shooting is the most important aspect of the game of basketball. With each player developing their own unique shooting style, the optimal release angle and jumping force differs from player to player. To determine these quantities, arm angle and jumping force were measured, until ten shots were made at each increasing distance. Arm angle was measured through the use of a Vernier goniometer strapped to a shooting players arm, while jumping force was calculated by a force plate beneath the players feet. After comparing shots taken at a variety of distances, there appears to be a positive linear correlation between angular velocity of the arm and distance from the basket. Furthermore, at a 90% confidence level, it was found that jumping force generally increased as distance from the basket increased. In spite of this, statistically significant differences between made and missed shots could only be identified through arm angular velocity.

Keywords: Basketball Shooting, Arm Kinematics, Release Angle, Jumping Force, Distance, Shot Selection

INTRODUCTION

Since 2009, a study found that the 3-point attempt rate in the NBA had risen from 0.222 to 0.377. Even more shockingly, it was found that the mid-range attempt rate, had decreased from 0.314 to 0.135, fading into relative nonexistence [1]. Additionally, increasing a team's three-point percentage by one point equated to 4.25 more wins in a season. [2] A further, more difficult shot had become more popular at basketball's highest level, making shooting an ever more important part of the game. The growing importance of taking deeper shots motivated a deeper study into players' shot mechanics and technique. A previous study by Okubo and Hubbard concluded that shots with higher mean, peak power values and average jump heights were more advantageous to players, providing them with maximum body extension. [3] However, this study was done by comparing player jumps without the ball and with the ball. The project provided little insight as to what exactly the optimal height a player should be jumping at, as well as whether or not the player jumping at maximum height would beneficial for shooting at different positions throughout on the court.

Ultimately, shot start and release angle contribute to the final trajectory of the ball while jump height determines the release point at which the ball begins. Physically, 45 degrees is the optimal angle at longer distances, however at closer distances where height becomes a priority over distance, players must modify their technique to reach the basket. [4]

In an attempt to quantify these changes, the arm angle and jump force adjustments made by players to compensate for increasing distance were examined. Shots were continuously each taken at varying distances across the court, ranging from 1 meter to 6 meters away (approximately three-point distance) from the rim until 10 shots were made. For each set of shots taken, arm angle and angular velocity through the shooter's shot motions were measured using a goniometer strapped to the players arm. In addition, the player's required jumping force was measured through the use of a force plate, placed strategically under each of the shooter's feet. Missed shots were also recorded, and all shots were taken directly in front of the rim without the use of the backboard. Scatter plots were constructed to help visualize the progression of force and angular velocity with respect to distance. Using this data, players may be able to more deeply analyze their shooting performance, by correcting their release angles, and accuracy. With this data, players can learn precisely what contributes to their performance on the court, and how to best position themselves to improve their abilities.

BACKGROUND AND THEORY

JUMP SHOT TECHNIQUE

Jump Shot technique varies from player to player and depends heavily on their background, gender, and dominant hand. Female players tend to prefer a two-handed shot, while their male counterparts prefer a one-handed shot. Left handed players typically perform the same process as right-handed players albeit mirrored to the opposite side. The process of a player's jump shot remains a unique part of each players game. However, the most common jump shot form begins in the dominant hand, slightly out in front of their forehead. As they progress through the shot, the rotate their shoulders, elbows, and wrist in quick succession to achieve the desired release. Jumping allows players to avoid being blocked and attain additional height on their shot. [5] Slightly changing the timing of each arm rotation dramatically changes the way a shot both looks and performs. As seen in frame 1-4 of Figure 1 below, the elbow is usually tucked underneath the ball for the entirety of the shot. A study by Hubbard found that a higher elbow technique diminished the effect of angular motion in the shoulder, simplifying the shooting technique. [5] Essentially, the starting position of your elbow limits a player's ability to rotate their arm throughout the motion, impeding their performance.

Figure 1: Typical Jump Shot Progression [6] The first image demonstrates the starting position of the jump shot, where the player is fully loaded for the shot motion. His feet are square to the basket, and the angle of his upper arm relative to his forearm is at its minimum. Meanwhile, the fourth represents the maximum arm angle, or the release point of the shot. Here, the player's arm is at full extension and he reaches the peak of his jump.

The two primary types of shots in the game of basketball, namely the set shot and the jump shot. A set shot occurs more frequently when a player is unguarded and unrushed. As a result, it is slower in its wrist and shoulder motions. During a set shot, players are either flat footed or at most take a light hop throughout the shot. On the other hand, the jump shot is less common, and refers to when a player jumps to his maximum height while releasing his or her shot. The release point of this type is higher but is more prone to lateral deviations. [7] These lateral deviations stem from last minute adjustments in response to the actions of a defending player. Despite these generalities, it's important to note that most of these qualities are highly dependent on player preference.

KINEMATICS OF SHOOTING

Previous studies have mapped the kinematics of a players shooting motion as a function of calculated angles. In a study by Okubo and Hubbard, the pair measured changes in three critical angles, the relationship between a player's upper arm relative to their body(Ψ_U), upper arm relative to their forearm (Ψ_F), and forearm relative to their wrist (Ψ_H) as seen in Figure 2. [8]

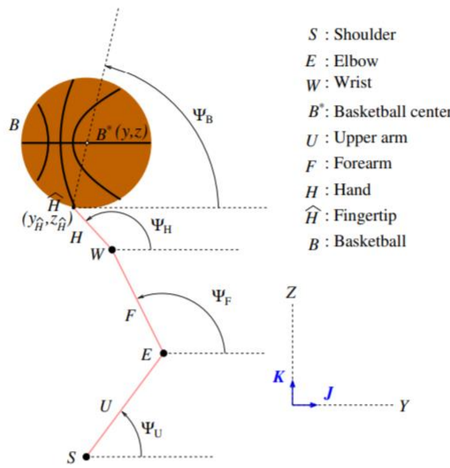

Figure 2: Kinematic Diagram of Arm Angles throughout a Players Jump Shot [8]

As a result of his studies, Okubo found that the release angular velocities and accelerations at the hand and the forearm were equal at the moment of release for shots of the same distance, and shoulder and elbow torques increase proportionally with distance. The calculated angle changes were 40, 82, and 115 degrees at release. [8] Based on this study, the release angular velocity and accelerations of the hand and the forearm are the same at the moment of release and the start (rest). Given this, the motion of the entire arm (excluding the shoulder) can be described by measuring only angular velocity of the elbow.

$$\tau = I\alpha$$

Furthermore, since the torque of the elbow increases with distance, and rotational inertia is constant, it can be said that the angular velocity of the elbow is changing since angular acceleration is nonzero.

In another study by Ercuji, where he tested three different players, he found that, as shot distance increases, the magnitude of the angular velocity of the elbow and shoulder increase, but the angular velocity of the wrist remains relatively the same. [7] The largest change in angular velocity and release angle occurred between the upper arm and forearm which contribute the most to the change in the ball's final velocity. Based on the significance of the elbow angle to jump shot performance, our study attempts to further generalize how this angle changes with distance.

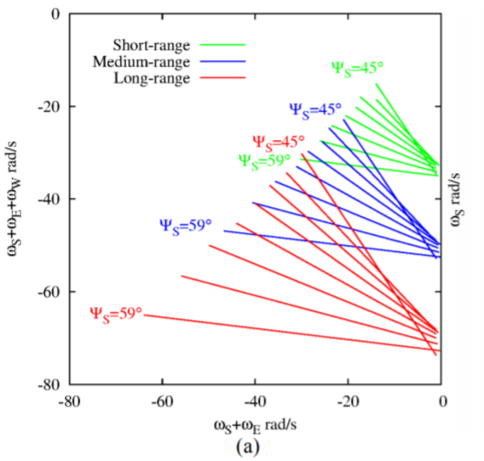

Figure 3: Comparison of Forearm Angle at Short, Medium, and Long Ranges [8] Short Range shots are denoted in Green Medium Range shots in Blue, and Long-Range Shots in Red. Each line represents a shot attempt. Ψ_S represents the sum of all the angles, (wrist, arm, elbow) of a player for that shot.

In addition, as seen in Figure 3 above, Okubo's paper concluded that total arm angle increased as shot distance increased, although it's important to note that this study was conducted with a different player at each range, with drastically different shot mechanics. Furthermore, since he graphed the collection of all the angles, it's difficult to determine which joints contributed the most to the motion itself. Given this, the current study aims to isolate the most critical of these angles, the elbow angle [see **Figure 4**].

Okubo did note that the lower the end release angle of the player, the higher the angular velocity of the player's arm, and the exit velocity of the ball. Essentially, it was found that depending on the final arm position of the shooter, the greater the velocity with which the ball is released a result that was also found in our study.

Regarding jump force, a previous study conducted by Struzik, found that reaction force depended heavily on player weight and height but found that the maximum ground reaction force was 5.57 multiplied by the player body weight. [3] For this experiment, that equates to approximately 1030 N. Generally, peak jump height was only optimal in the case without an arm swing jump shot. Struzik found that players who depended less on the rotation of their arm compensated by increasing the height of their jump. To further explore this connection, this study aims to determine the optimal height while progression through the shot motion and its codependence on arm angle.

EXPERIMENTAL DESIGN

APPARATUS/SENSOR CONFIGURATION

In order to determine the change in arm angle throughout the shot, a Vernier goniometer was strapped to the player's arm for the duration of the shot. The goniometer has a range of 0 to 340 degrees and a calculated resolution of 0.12 degrees. As the player took their shot, they would jump and land on a force plate situated beneath their feet. The force plate operates at a force range of -850 to 3500 N with a resolution of 1.2 N. Shots were continuously taken until 10 were made each from one to six meters. The designated starting position (minimum angle) is shown below in Figure 4.

Figure 4: The Experimental Setup, Diagram of Sensors with respect to the player. Goniometer strapped to player's arm. Designated Starting Position of each shot (i.e Minimum Angle). Measured angle theta shown in purple.

In an effort to limit noise and other pre-shot movements, the player would start in a fixed position in preparation of his or her shot. This mirrors the start position of a catch and shoot jump shot, a shot common in regular play. After following through, the player would hold their arm up at max extension, in an effort to keep their arm as still as possible after the ball has left his/her hand. An additional person to help with data collection allowed for data to be collected without disturbing the shot.

EXECUTION OF JUMP SHOT

For each jump shot, a player began each shot on top of a Force Plate (see Figure 5). The player begins in a set position and proceeds to move up through their shot motion. After releasing the ball, the player lands directly onto the force plate, and holds his arm up at full extension after the shot. The change in angle over time and applied force on the force plate allowed for analysis as a function of time and distance. Data was collected at 50 Hz over 10 seconds, through a Lab Quest Mini.

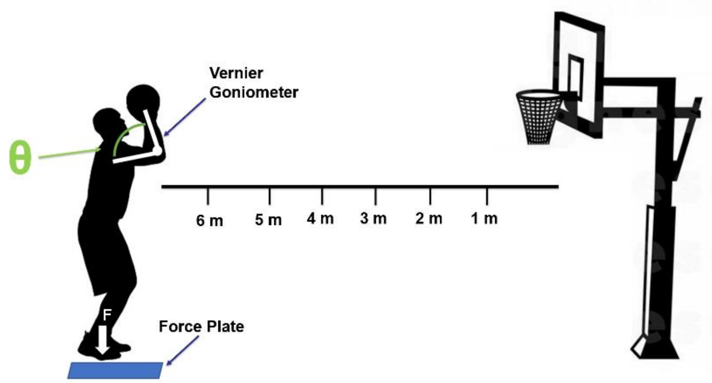

Figure 5: Experimental Setup Diagram, measured angle theta displayed in green. Shots were continuously taken at each distance until ten were made at each position. Measured Jump Force points downward into the Force Plate

ANALYSIS METHODS

After collecting the data, further analysis was conducted on each attempted shot. For each trial (attempted shot), the Start Angle, End Angle, and Max Jump Force locations were isolated, from Raw Logger Pro Data. Through the use of MATLAB, maximum angular speed was computed using its built-in gradient function on the collected arm angle data. By separating these critical points in time, they could be used for further analysis later on.

Shortly Afterwards, Start Angle, End Angle, Change in Arm Angle, Angular Velocity, and Max Jump Force as a function of distance were plotted on a scatter plot. After noticing a correlation in the graphs of angular velocity and change in arm angle, MATLAB's cftool was used to test, exponential and quadratic fits for a potential best fit. Prediction bounds for each were also graphed to clearly identify significance. For jumping force various t-tests were conducted between made shots at different distances to determine confidence levels in noted increases in jumping force and distance (built-in ttest2 function). Finally, t-tests were once again conducted between made and missed shots at the same distances for angular velocity and jumping force to determine whether made or missed shots could be distinguished purely through jumping force and angular velocity.

RESULTS AND DISCUSSION

In order to begin investigating the correlation between distance, arm angle and jumping force, raw angular motion and force generation data was collected through the use of a Vernier goniometer and force plates. The collected raw data was then interpreted to determine the key moments in time throughout the jump shot, with which analysis was conducted.

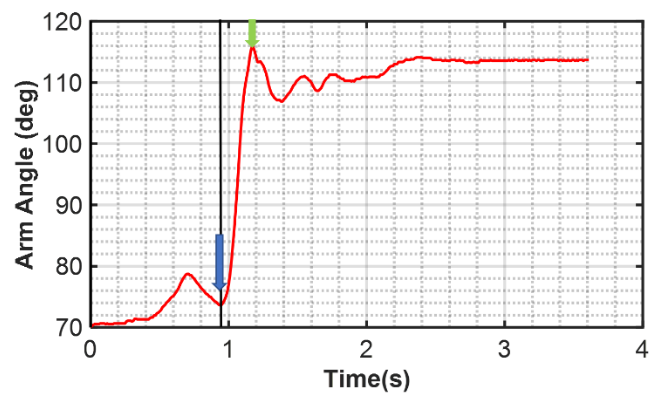

Figure 6: This plot demonstrates the relationship between arm angle and time for a single jump shot at a distance of 3 meters. Arm angle was collected through the use of a Vernier Goniometer. The blue arrow denotes the starting (minimum) angle of the jump shot, the position at which the arm is fully cocked back [**see Figure 1, Slide 2 for reference**]. The green arrow represents the end angle of the jump shot or the position at which the arm is at full extension [**see Figure 1, Slide 4 for reference**]. The vertical black line denotes the beginning of the jump shot motion.

As seen in Figure 6, the start angle, denoted with the blue arrow was defined as the minimum angle throughout the jump shot. Typically, as a person progresses through the jump shot motion, the point at which the angle between the elbow and upper arm is at its minimum represents the start of the shot. [**See Figure 1, Slide 2**] Conversely, the green arrow represents the end angle or the point at which the player's arm is at full extension [**Figure 1, Slide 4**]. It becomes apparent that once the start angle is reached, the arm of the player begins to accelerate as seen with the orange arrow in Figure 7. Figure 7 was obtained by taking the derivative of Figure 6 using MATLAB's gradient function.

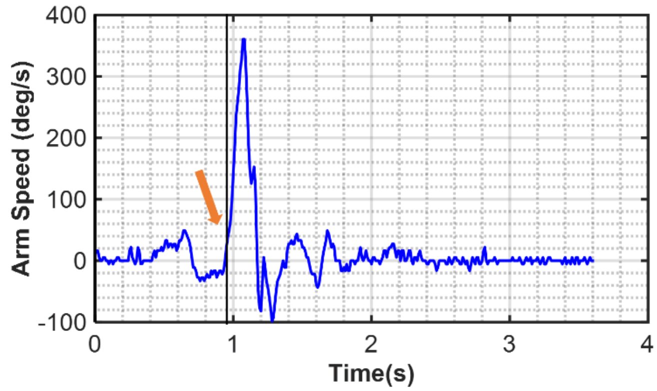

Figure 7: This plot demonstrates the relationship between arm speed and time for a single jump shot at a distance of 3 meters. The arm speed was determined by taking the gradient of the arm angle over time (Figure 5). The orange arrow represents the start of the shot, at a time synonymous with the start angle. Again, the vertical black line denotes the beginning of the jump shot motion.

The pattern extends to jump force, where the maximum denotes the peak force exerted into the ground during the jump. This point was defined as the max force and was used as the primary point of analysis.

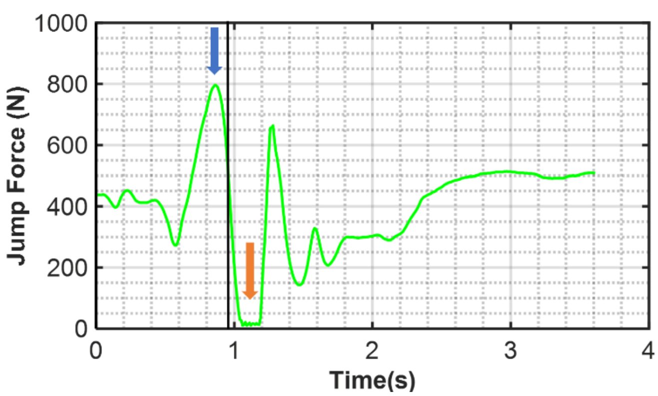

Figure 8: This plot demonstrates the relationship between jump force and time for a single jump shot at a distance of 3 meters. The blue arrow denotes the maximum force exerted by the player on the ground throughout his jump. The subsequent dip, denoted by the orange arrow reflects the time in which the player is in the air, and exerts no force on the force plate. At around 1.2 seconds the player lands, and the Jump Force begins to increase.

After collecting the raw data, it was subsequently interpreted over the distances in which they were taken. A scatterplot was used to display their respective relationship with distance, for each shot and a fit line used to define that relation.

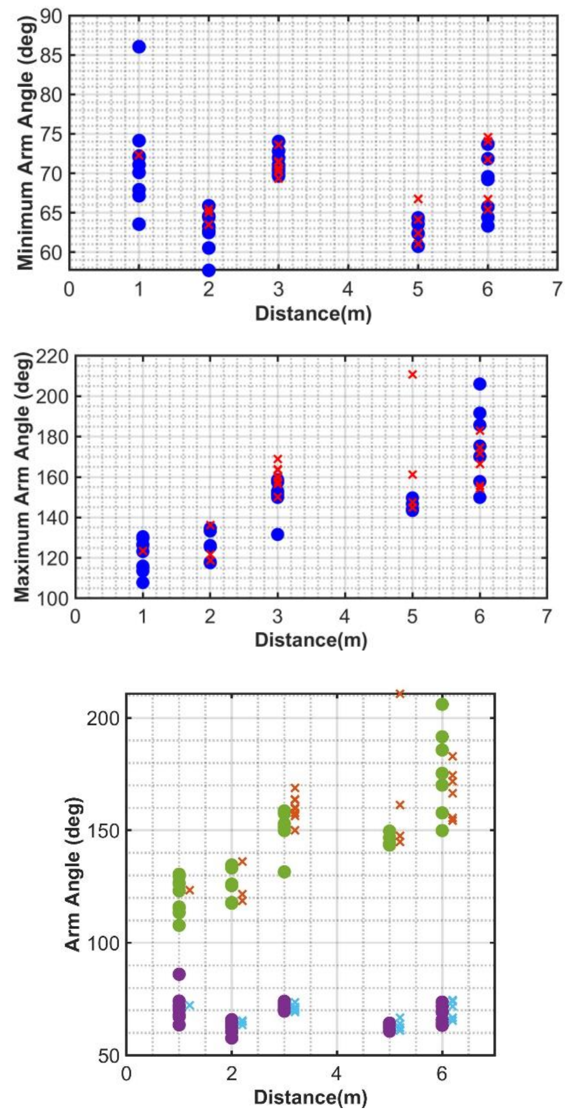

Figure 9: These first two graphs demonstrate the respect minimum start angle [**See Figure 1 Slide 2**] and maximum angle [**See Figure 1 Slide 4**] with respect to distance. After conducting a t-test, there was no significant difference in minimum angle with respect to distance. However, maximum arm angle appears to increase with distance. In the bottom graph, the two are plotted together. (Purple and Blue represent made and missed minimum angles, while Green and Orange represent made and missed maximum angles respectively) It becomes increasingly apparent that relative to the maximum angle, the minimum angle stays relatively constant across distance. *Note: Missed Shots were displaced to the left for ease of viewing. Missed Shots are denoted with x, while Made Shots are denoted with circles.*

As seen in Figure 9, start angle remained relatively constant across different distances, and after conducting a t-test, there appeared to be no significant increases in minimum start angle as distance progressively increase.

On the other hand, the opposite was found for end angle as it significantly changed almost linearly with distance.

Afterwards, minimum arm angle was subtracted from maximum arm angle to find the total change in arm angle for each shot. The results are seen in Figure 10 below.

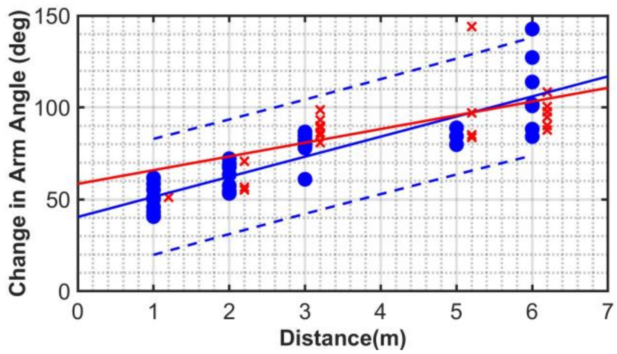

Figure 10: This graph represents the change in arm angle with respect to distance. Change in arm angle was defined as the difference between end angle and start angle. A linear fit line for made shots is displayed in blue with the equation $\Delta\theta = p1 \times d + p2$, where $p1 = 10.9 \pm 2.2, p2 = 40.44 \pm 7.7$. 95% confidence predictions lines are displayed with the blue dashed line. A linear fit line for missed shots is displayed in red with the equation $\Delta\theta = p1 \times d + p2$, where $p1 = 7.5 \pm 4.2, p2 = 58 \pm 18$. *Note: Missed Shots were displaced to the left for ease of viewing. Missed Shots are denoted in red, while Made Shots are denoted in blue.*

As seen in Figure 10, change in arm angle was shown to increase linearly with distance. As the majority of the data lies within the prediction bounds, the data conforms with the fit with 95% confidence. According to the fit for every additional meter of distance, a player extends 10.9 additional degrees to compensate. Furthermore, this trend also applied to missed shots. Although not show above, the data conformed to the 95% confidence interval. This trend further extended into the progression of player arm speed at different distances.

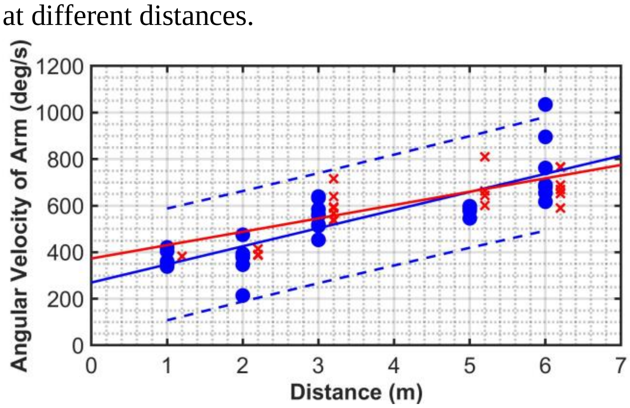

Figure 11: This graph represents the angular velocity of the arm with respect to distance. A linear fit line for made shots is displayed in blue with the equation $\omega = p1 \times d + p2$, where $p1 = 78 \pm 16 \left(\frac{\circ}{s}\right), p2 = 270 \pm 58 \left(\frac{\circ}{s}\right)$. 95% confidence predictions lines are displayed with the blue dashed line. A linear fit line for missed shots is displayed in red with the equation $\omega = p1 \times d + p2$, where $p1 = 57 \pm 22 \left(\frac{\circ}{s}\right), p2 = 372 \pm 94 \left(\frac{\circ}{s}\right)$. *Note: Missed Shots were displaced to the left for ease of viewing. Missed Shots are denoted in red, while Made Shots are denoted in blue.*

Player arm speed also increased with linearly with distance. Physically this makes sense, as a higher angular velocity enables a player to send the ball further through the air. More interestingly, however, there appeared to be no statistically significant difference between the arm speed for made and missed shots at the same distance. Both demonstrate a significant increase over distance but made and missed shots at the same distances were only distinguishable through the differing slopes in the data.

An additional t-test was conducted for both made and missed shots at similar distances. However, it was found that at the 95% confidence interval, none demonstrated a statistically significant difference The closest was at 6 meters, with a p-value of 2.36 much greater than the desired threshold of 0.05.

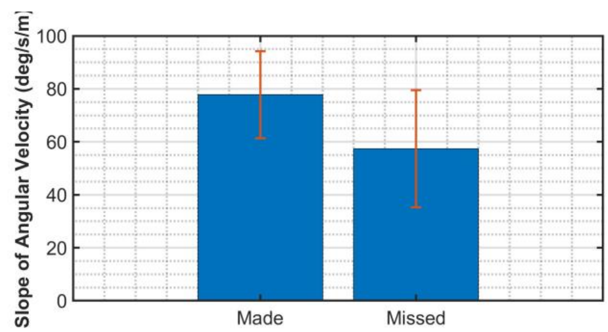

Figure 12: This graph compares the slopes of the linear fits for made and missed shots respectively. Error bars are shown in orange.

Regarding player jump forces, maximum jump force was plotted as a function of distance. Instead of conducting parametric fitting, a t-test for two samples assuming unequal variance were taken to compare made shots at different distances. Ultimately, the t-test found that there was a statistically significant difference in jump force between shots taken at three and six meters. The calculated hypothetical mean difference was 660 N, at a 95% confidence level. ($p = 0.05$). While there wasn't a statistically significant difference (95%) in jump force for shots taken between one and three meters ($p = 0.19$), there was a significant increase at the 90% confidence level, as seen in Figure 13. From one to six meters, the difference is more profound, with a p-value of 6.84e-06, much smaller than the threshold 0.05. Ultimately, based on the initial gathered data, it can be said with 90% confidence, that max jump force increases as distance from the rim increases.

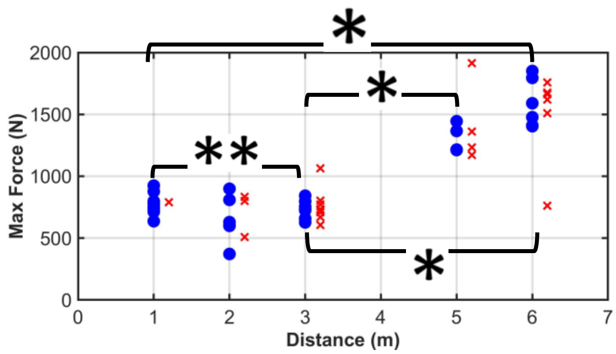

Figure 13: This graph compares the maximum jumping force a player exerts on the ground with distance. The bracket with one star denotes a statistically significant difference at the 95% confidence level, while the bracket with two stars denotes a statistically significant difference at the 90% confidence level. *Note: Missed Shots were displaced to the left for ease of viewing. Missed Shots are denoted in red, while Made Shots are denoted in blue.*

Additionally, a second t-test was taken to determine whether there was correlation between made and missed shots at similar distances. At 3 meters, a t-test between made and missed shots found a p value of 0.52, much larger than the threshold of 0.05. Similarly, at 6 meters, p was found to be 0.71. As a result, at the 95% confidence level there is no significant difference in jump force between made and missed shots at a given distance.

CONCLUSIONS

Preliminary analysis of the data demonstrates no statistically significant increase in minimum angle, however, the results show significant increases in maximum arm angle with distance. This makes sense especially when one considers the shooting mechanics of a player. At the start of each players jump shot, the initial position of the shooter remains the same, regardless of their position on the court. This reflects accurately with the data as there appears to be no correlation between minimum angle and distance. The scenario changes when one considers the progression of a player's maximum angle. If the minimum stays relatively constant for each shot, the maximum angle appears to compensate. For instance, if a player requires less force (at closer distances), he or she will cut their shot short, failing to reach full extension. In other words, players only fully progress through their shooting motion when necessary.

Analysis of the data also suggests that change in arm angle and arm angular velocity for made and missed shots increase linearly as a function of distance. For made shots, the angular velocity increased by 78 deg/m-sec, while for missed shots the angular velocity increased by 57 deg/m-sec. Here, the angular velocity for made shots increased by about 21 deg/sec-m more than missed shots did. Furthermore, as a result of various t-tests, there are no statistically significant differences between the peak angular velocity for made and missed shots at the same distance, suggesting that the shots that were missed were not a direct result of the angular velocity.

Ultimately, it can also be said that there is a statistically significant increase in jumping force from 1-6, 3-6, 3-5 meters at the 95 % confidence interval. At the 90% confidence level, it can also be concluded that jumping force increases from 1-3 meters. The peak jump force at 6 meters was nearly double that of the results at 1 meter. Made and missed shots also seem to be independent of jump force as t-tests found no significant difference in these values all of distances. Essentially, the results of the t-tests for made and missed shots suggest that their respective jump forces are indistinguishable at the same distance.

In the future, additional investigation is required into what goes on at extended distances beyond six meters. Furthermore, an increase in the number of missed and made shots may introduce further conclusions and distinctions that can be made. Introducing angled shots, the use of the backboard, and mapping a different shoulder/arm joint may provide additional insight in the kinematics of a jump shot. Finally, after noting the smaller change in angular velocity over distance, for missed shots, another avenue of exploration could be to see if said shots are falling short of the rim or overshooting all together.

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